Limit of a Function: The notation means that as x approaches the value *a,* then f(x) approaches the number L.

Epsilon-Delta Notation: A notation for limits where one specifies for the x-range of the limit (notated by δ), that the y-values will be within a range (notated by ε), this is such that that the following inequalities may be defined: 0 < |x-a| < δ, |f(x) – L| < ε.

Left-Hand and Right-Hand Limits: The notation means that as you approach x from the left, the value of f(x) approaches L and is referred to as a left-handed limit. Meanwhile, means that you approach x from the right, the value of f(x) approaches L and is referred to as a right-handed limit. These limits are important when considering things such as absolute value functions, where the value may differ depending on which direction one comes from. Notably, if both the left-handed and right-handed limits of *a* are equal, then the limit of f(x) as it approaches *a* is equal to L.

Note (Abbreviation): DNE in the context of limits implies the limit does not exist.

Definition of Continuity at a Point: A function *f* is considered continuous at *c* when-

1. f(x) is defined
2. exists

Properties of Limits: Given real numbers b and c, positive number n and functions f and f with limits and , we may find the following:

Constant Property: If the equation of f(x) is a constant the limit of any value of c, will equal the constant.

Scalar Property: Given scalar b,

Sum/Difference Property:

Product Property:

Quotient Property: , if k ≠0.

Power Property: , if n is a positive integer or if L > 0

Numerical Approach to Finding Limits: Given that the limit for f(x) exists as it approaches c, one may find the limit of f(x) by computing the values close to f(c) and approximating the limit of f(c).

Transcendental Functions: Functions that are not rational functions or polynomials. Examples include sin(x) and ln(x).

Squeeze Theorem: Given the inequality f(x) ≤ g(x) ≤ h(x), if f(x) = h(x), then f(x) = g(x) = h(x). This may also be used in application to limits i.e. if and , then .

Rationalising Discontinuities: If a rational function is in the form , one may find the limit of where R(x) is discontinuous by simplifying such that the discontinuity is removed in the resulting fraction. Note the resulting fraction is equivalent to R(x) except where the original rational function is undefined.

Positive Interval Theorem: If and L > 0, then there is an open interval (c – δ, c + δ) containing c such that f(x)>0 for every x in (c – δ, c + δ) except possibly at x = c. The same may be said if L is negative.

Linear Theorem of Limits: Given the linear function f(x) = x, we can say that for any value of c, the limit of f(x) will equal c. In other terms, .

Powers of the Limit of f(x): If n is a positive rational number and exists, then .

Substitution Theorem of Limits: If f is a function and a is within the domain of f, then .

Substitution Theorem of Limits of Radical Functions: if c > 0 and n is a positive integer, or if a ≤ 0 and n is an odd integer, then .

Rational Infinity Theorem: If k is a positive rational number and c is any real number, then , provided is always defined.

Types of Discontinuities: There are 3 types of discontinuities shown below-

1. Removable: A discontinuity that may be removed by defining f(c) appropriately.
2. Jump: A discontinuity that appears to jump from one value to another
3. Infinite: A discontinuity involving infinity.

Continuity in Polynomial and Rational Functions: A polynomial function f is continuous at every real number, while a rational function is continuous at every number except where q(x) = 0.

Composition and Combination of Continuous Functions: If f and g are continuous at c, then the following are also continuous at c-

I:(f+g) II: (f-g) III: (f\*g) IV: (f/g), g(c) ≠ 0.

In addition, if and if f is continuous at b, then .

Intermediate Value Theorem: If f is continuous on close interval [a, b] and if w is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = w.

No Zero Theorem: If a function f is continuous and has no zeros on an interval, then either f(x) > 0 or f(x) < 0 for every x on that interval.

Note: Some useful limits to remember are , , .

Proof of : Given the diagram shown to the right, we can determine that the triangle ABO has an area of (its height is sin(Ø)), we can also determine the Sector AB has an area of () and the triangle ADO has an area of (its height is tan(x)). After finding the areas of these triangles we can order then in an inequality, || < | | < ||. We can simplify this inequality by multiplying it by || to get 1 < < . We can then invert the inequality and get 1 > > cos(x), since cos(x) moves towards 1 the closer x is to 0 we can then use the squeeze theorem to say that the .

Derivative: The instantaneous rate of change of a variable with respect to another variable. The derivative of a function shows the rate of change of the input x relative to its output f(x).

Derivative of f(x): A function, f(x), has a derivative at x = c, if exists and is finite. The limit is called the derivative of f(x) at x and is denoted by f’(x). The function of the limit is to find the rate of change of f(x) by using the general formula of the rate of change over an interval and reducing that length of the interval to 0 with the interval length represented by h.

Continuity and Derivatives: If a function f has a derivative at a point c, then f is continuous at c.

Note: Differentiability of a function and the existence of a derivative at a point are only necessarily equivalent with single variable functions.

Tangent Line of a Point on a Function: The tangent line of a point on a function f(x) at P(c, f(c)) is the line through P with slope f’(c). Its equation can be given by y – f(c) = f’(c)(x-a). If f’(c) does not exist, then the tangent line at c does not exist.

Notation of Derivative: If y = f(x), then the derivative can be written .

Formulae for Derivative:

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